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NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M. (Princeton); Ph. D. (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from March Number.]

PROPOSITION XXXIV, *in which is investigated a certain curve arising from the hypothesis of acute angle. [An equidistantial of a straight has its chords between it and the straight.]*

Let the straight CD join equal perpendiculars AC, BD standing upon any straight AB. Then AB, CD being bisected in the points M and H (Fig. 42.), MH is joined perpendicular (by Proposition II) to each. Again in this hypothesis the angles at the join CD are supposed acute. Therefore in the quadrilateral $AMHC$ (by Corollary I after Proposition III) MH will be less than AC. Hence now, if in MH produced MK be taken equal to AC, the points C, K, D pertain to the curve here investigated. Then the angles at the join CK will be themselves acute (by Proposition VII).

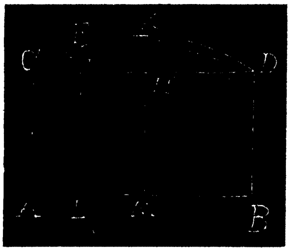

Fig. 42.

Therefore the join LX, which bisects, and therefore (by Proposition II), is at right angles to AM, CK, will be likewise (by Corollary I after Proposition III) less than AC. Wherefore, if in LX produced we assume LF equal to AC or MK, the point F also will pertain to this curve. Further, joining CF, and

FK we find likewise two other points pertaining to the same curve. And so on for ever.

But what I say for finding points between the points *C* and *K*, the same also holds good uniformly for finding points between the points *K* and *D*. Obviously the curve *CKD*, arising from the hypothesis of acute angle, is the line joining the extremities of all equal perpendiculars erected upon the same base toward the same part, which assuredly can come under the name ordinates. It is, I add, a line of such sort, that on account of the hypothesis of acute angle, from which it arises, it always is concave toward the parts of the opposite base *AB*. Quod quidem hoc loco declarandum, ac demonstrandum a nobis erat.

PROPOSITION XXXV. *If from any point L of the base AB the ordinate LF is drawn to this curve CKD: I say the straight NFX perpendicular to LF must on both sides fall wholly toward the convex parts of this curve, and therefore it will be tangent to this curve.*

Proof. For if possible, let a certain point *X* (Fig. 43.) of *NFX* fall within the cavity of this curve. Let fall from the point *X* to the base *AB* the perpendicular *XP*, which prolonged through *X* meets the curve in a certain point *R*. Now thus. In the quadrilateral *LFXP* the angle at the point *X* will be neither right nor obtuse: else (Proposition V and Proposition VI) would be destroyed the present hypothesis of acute angle.

Therefore the aforesaid angle will be acute. Wherefore (from Corollary I after Proposition III) *PX* and so much more *PR* will be greater than *LF*. But this is absurd (from the preceding) against the nature of this curve.

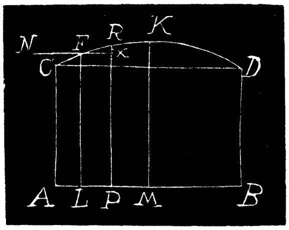

Fig. 43.

So *NF* produced must fall wholly toward the convex parts, and so it will be tangent to this curve. Quod erat demonstrandum.

[To be Continued.]

SUMMATION OF SERIES.

By G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

$$\frac{\sin h\theta}{\theta}=\left(1+\frac{\theta^2}{\pi^2}\right)\left(1+\frac{\theta^2}{2^2\pi^2}\right)\left(1+\frac{\theta^2}{3^2\pi^2}\right)\left(1+\frac{\theta^2}{4^2\pi^2}\right)\ldots\ldots$$

$$=1+\frac{\theta^2}{3\,!}+\frac{\theta^4}{5\,!}+\frac{\theta^6}{7\,!}+\frac{\theta^8}{9\,!}+\ldots\ldots(A).$$

$$\therefore\ \log\left(1+\frac{\theta^2}{\pi^2}\right)+\log\left(1+\frac{\theta^2}{2^2\pi^2}\right)+\log\left(1+\frac{\theta^2}{3^2\pi^2}\right)+\log\left(1+\frac{\theta^2}{4^2\pi^2}\right)\ldots\ldots$$